10.8 Outlet Hydraulics

10.8.1 General

A stage-discharge (performance) curve defines the relationship between the depth of water and the discharge or outflow from a storage facility. A typical storage facility will have both a principal and an emergency outlet. The principal outlet is usually designed with a capacity sufficient to convey the design flood without allowing flow to enter the emergency spillway. The structure for the principal outlet will typically consist of a pipe culvert, weir, orifice, or other appropriate hydraulic control device. Multiple outlet control devices are often used to provide discharge controls for multiple frequency storms.

Development of a composite stage-discharge curve requires consideration of the discharge rating relationships for each component of the outlet structure. The following sections present design relationships for typical outlet controls.

10.8.2 Orifices

For a single orifice as illustrated in Figure 10-11 (a), orifice flow can be determined using equation 10.17.

$$Q = C_o A_o (2 g H_o)^{0.5} \qquad \textbf{(10.17)}$$

where:
- Q = the orifice flow rate, m^3/s (ft^3/s)
- C_o = discharge coefficient (0.40 - 0.60)
- A_o = area of orifice, m^2 (ft^2)
- H_o = effective head on the orifice measured from the centroid of the opening, m (ft)
- g = gravitational acceleration, 9.81 m/s^2 (32.2 ft/s^2)

If the orifice discharges as a free outfall, then the effective head is measured from the centerline of the orifice to the upstream water surface elevation. If the orifice discharge is submerged, then the effective head is the difference in elevation of the upstream and downstream water surfaces. This latter condition of a submerged discharge is shown in Figure 10-11(b).

For square-edged, uniform orifice entrance conditions, a discharge coefficient of 0.6 should be used. For ragged edged orifices, such as those resulting from the use of an acetylene torch to cut orifice openings in corrugated pipe, a value of 0.4 should be used.

For circular orifices with C_o set equal to 0.6, the following equation results:

$$Q = K_{or} D^2 H_o^{0.50} \qquad \textbf{(10.18)}$$

where:
- K_{or} = 2.09 in S.I. units (3.78 in English units)
- D = orifice diameter, m (ft)

Pipes smaller than 0.3 m (1 ft) in diameter may be analyzed as a submerged orifice as long as H_o/D is greater than 1.5. Pipes greater than 0.3 m (1 ft) in diameter should be analyzed as a discharge pipe with headwater and tailwater effects taken into account, not just as an orifice.

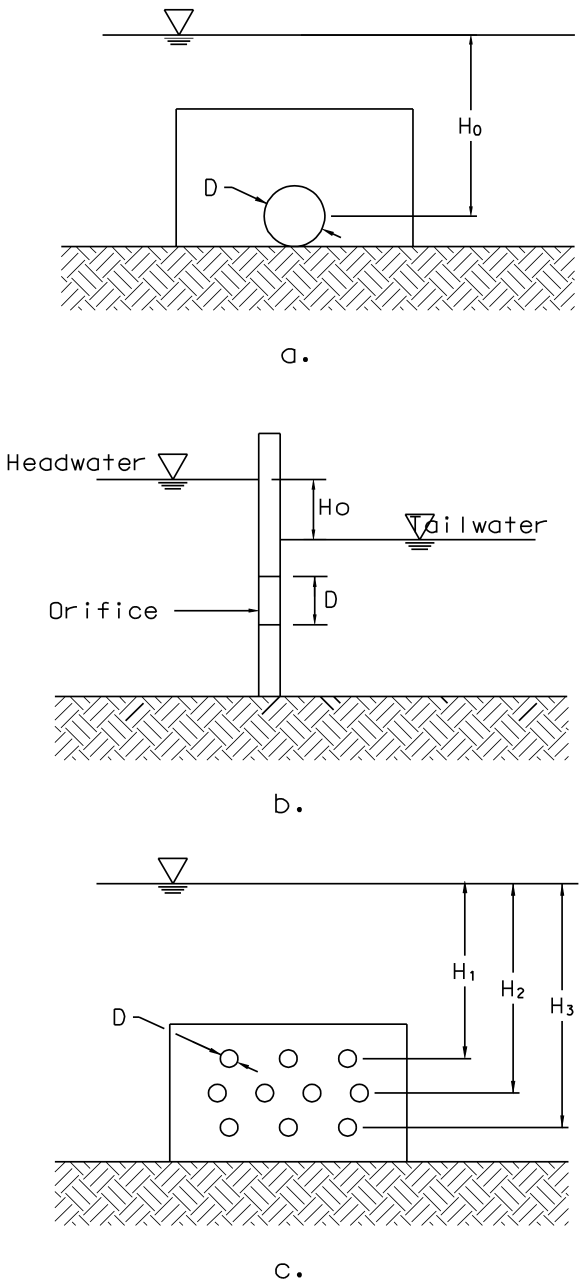

Figure 10-11 Definition Sketch for Orifice Flow

Flow through multiple orifices (see Figure 10-11(c)) can be computed by summing the flow through individual orifices. For multiple orifices of the same size and under the influence of the same effective head, the total flow can be determined by multiplying the discharge for a single orifice by the number of openings. The procedure is demonstrated in the following example:

Example 10-4

Given: *Given the orifice plate in Figure 10-11 (c) with a free discharge and:*

orifice diameter	=	*25 mm (1.0 in)*
H_1	=	*1.1 m (3.6 ft)*
H_2	=	*1.2 m (3.9 ft)*
H_3	=	*1.3 m (4.3 ft)*

Find: *Total discharge through the orifice plate.*

Solution: *Using a modification of equation 10.18 for multiple orifices,*

$$Q_i = K D^2 (H_i)^{0.5} N_i$$
$$Q_i = (2.09)(0.025)^2 (H_i)^{0.5} N_i = 0.0013 H_i^{0.5} N_i$$

$$Q_1 = 0.0013 (1.1)^{0.5} (3) = 0.0040$$
$$Q_2 = 0.0013 (1.2)^{0.5} (4) = 0.0058$$
$$Q_2 = 0.0013 (1.3)^{0.5} (3) = 0.0045$$

$$Q_{total} = Q_1 + Q_2 + Q_3 = 0.0143 \text{ m}^3/\text{s} \ (0.50 \text{ ft}^3/\text{s})$$

Example 10-5

Given: *Given the circular orifice in Figure 10-11(a) with:*

orifice diameter = 0.15 m (0.5 ft)
orifice invert = 10.0 m (32.8 ft)
discharge coeff. = 0.60

Find: *The stage - discharge rating between 10 m (32.8 ft) and 12.0 m (39.4 ft).*

Solution: *Using equation 10.17 with D = 0.15 m yields the following relationship between the effective head on the orifice (H_o) and the resulting discharge:*

$$Q = 0.047\, H_o^{0.5}$$
$$H_o = Depth - D/_2$$

The solution of this equation in table form is as follows:

Stage Discharge Tabulation

DEPTH		*STAGE*		*DISCHARGE*	
(meters)	*(feet)*	*(meters)*	*(feet)*	*(m^3/s)*	*(ft^3/s)*
0.00	*0.0*	*10.0*	*32.8*	*0.000*	*0.00*
0.20	*0.7*	*10.2*	*33.5*	*0.011*	*0.37*
0.40	*1.3*	*10.4*	*34.1*	*0.024*	*0.83*
0.60	*2.0*	*10.6*	*34.8*	*0.032*	*1.11*
0.80	*2.6*	*10.8*	*35.4*	*0.038*	*1.34*
1.00	*3.3*	*11.0*	*36.1*	*0.043*	*1.53*
1.20	*3.9*	*11.2*	*36.7*	*0.048*	*1.70*
1.40	*4.6*	*11.4*	*37.4*	*0.053*	*1.85*
1.60	*5.2*	*11.6*	*38.0*	*0.057*	*2.00*
1.80	*5.9*	*11.8*	*38.7*	*0.061*	*2.13*
2.00	*6.6*	*12.0*	*39.4*	*0.064*	*2.26*

10.8.3 Weirs

Relationships for sharp-crested, broad-crested, V-notch, and proportional weirs are provided in the following sections:

Sharp Crested Weirs

Typical sharp crested weirs are illustrated in Figure 10-12. Equation 10.19 provides the discharge relationship for **sharp crested weirs** with no end contractions (illustrated in Figure 12a).

$$Q = C_{scw}\ L H^{1.5} \qquad \textbf{(10.19)}$$

where:
- Q = discharge, m^3/s (ft^3/s)
- L = horizontal weir length, m (ft)
- H = head above weir crest excluding velocity head, m (ft)
- C_{SCW} = $1.81 + 0.22\ (H/H_c)$ [$3.27 + 0.4\ (H/H_c)$ in English units]

As indicated above, the value of the coefficient C_{SCW} is known to vary with the ratio H/H_c (see Figure 10-12c for definition of terms). For values of the ratio H/H_c less than 0.3, a constant C_{SCW} of 1.84 (3.33 in English units) is often used.

Equation 10.20 provides the discharge equation for **sharp-crested weirs with end contractions** (illustrated in Figure 10-12(b)). As indicated above, the value of the coefficient C_{scw} is known to vary with the ratio H/H_c (see Figure 10-13c for definition of terms). For values of the ratio H/H_c less than 0.3, a constant C_{scw} of 1.84 (3.33 in English units) is often used.

$$Q = C_{SCW}\ (L - 0.2\ H)\ H^{1.5} \qquad \textbf{(10.20)}$$

Sharp-crested weirs will be effected by submergence when the tailwater rises above the weir crest elevation, as shown in Figure 10-12(d). The result will be that the discharge over the weir will be reduced.

The discharge equation for a **submerged sharp-crested weir** is:

$$Q_s = Q_r\ (1 - (H_2 / H_1)^{1.5})^{0.385} \qquad \textbf{(10.21)}$$

where:
- Q_s = submerged flow, m^3/s (ft^3/s)
- Q_r = unsubmerged weir flow from equation 10.19 or 10.20, m^3/s (ft^3/s)
- H_1 = upstream head above crest, m (ft)
- H_2 = downstream head above crest, m (ft)

Flow over the top edge of a riser pipe is typically treated as flow over a sharp crested weir with no end constrictions. Equation 10.19 should be used for this case.

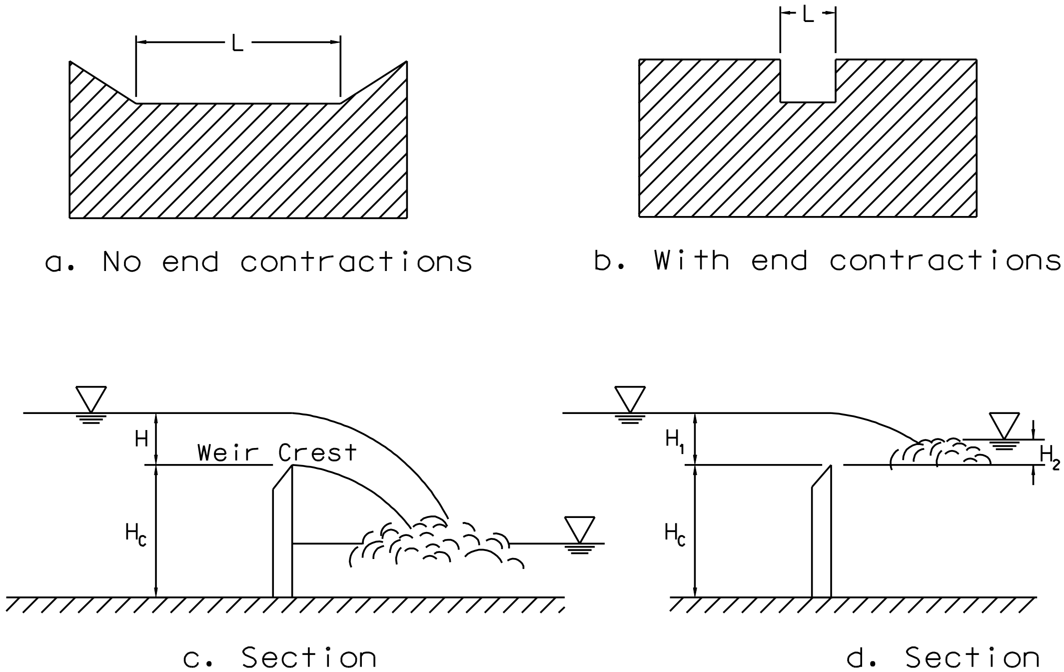

Figure 10-12 Sharp Crested Weirs

Example 10-6

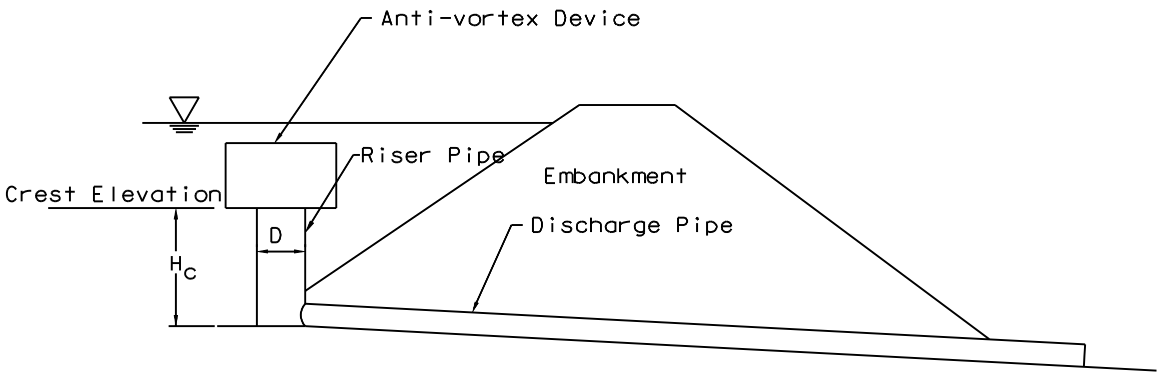

Figure 10-13 Riser Pipe

Given: *A riser pipe as shown in Figure 10-13 with the following characteristics:*

diameter (D) = 0.53 m (1.75 ft)
crest elevation = 10.8 m (35.4 ft)
weir height (H_c) = 0.8 m (2.6 ft)

Find: *The stage - discharge rating for the riser pipe between 10 m (32.8 ft) and 12.0 m (39.4 ft).*

Solution: *Since the riser pipe functions as both a weir and an orifice (depending on stage), the rating is developed by comparing the stage - discharge produced by both weir and orifice flow as follows:*

Using equation 10.18 for orifices with D = 0.53 m (1.75 ft) yields the following relationship between the effective head on the orifice (H_o) and the resulting discharge:

$$Q = K_{or} D^2 H_o^{0.50}$$
$$Q = (2.09)(0.53)^2 H_o^{0.50}$$
$$Q = 0.587 H_o^{0.50}$$

Using equation 10.19 for sharp crested weirs with C_{SCW} = 1.84 (H/H_c assumed less than 0.3), and L = pipe circumference = 1.67 m (5.5 ft) yields the following relationship between the effective head on the riser (H) and the resulting discharge:

$$Q = C_{SCW} L H^{1.5}$$
$$Q = (1.84)(1.67) H^{1.5}$$
$$Q = 3.073 H^{1.5}$$

The resulting stage - discharge relationship is summarized in the following table:

STAGE		*EFFECTIVE HEAD*		*ORIFICE FLOW*		*WEIR FLOW*	
(m)	*(ft)*	*(m)*	*(ft)*	*(m^3/s)*	*(ft^3/s)*	*(m^3/s)*	*(ft^3/s)*
10.0	*32.8*	*0.0*	*0.0*	*0.00*	*0.0*	*0.00*	*0.0*
10.8	*35.4*	*0.0*	*0.0*	*0.00*	*0.0*	*0.00*	*0.0*
10.9	*35.7*	*0.1*	*0.3*	*0.19*	*6.6*	*0.10*	*3.4*
11.0	*36.1*	*0.2*	*0.7*	*0.26*	*9.2*	*0.27*	*9.5*
11.2	*36.7*	*0.4*	*1.3*	*0.37*	*13.1*	*0.78*	*27.5*
11.4	*37.4*	*0.6*	*2.0*	*0.45*	*15.9*	*1.43*	*50.5*
11.6	*38.1*	*0.8*	*2.6*	*0.53*	*18.7*	*2.20*	*77.7*
11.8	*38.7*	*1.0*	*3.3*	*0.59*	*20.8*	*3.07*	*108.4*
12.0	*39.4*	*1.2*	*3.9*	*0.64*	*22.6*	*4.04*	*142.7*

Designates controlling flow.

The flow condition, orifice or weir, producing the lowest discharge for a given stage defines the controlling relationship. As illustrated in the above table, at a stage of 10.9 m (35.7 ft) weir flow controls the discharge through the riser. However, at and above a stage of 11.0 m (36.1 ft), orifice flow controls the discharge through the riser.

Broad-Crested Weir

The equation typically used for a broad-crested weir is:

$$Q = C_{BCW}\, L\, H^{1.5} \qquad \textbf{(10.22)}$$

where:
- Q = discharge, m^3/s (ft^3/s)
- C_{BCW} = broad-crested weir coefficient, 1.44 - 1.70 (2.61 to 3.08)
- L = broad-crested weir length, m (ft)
- H = head above weir crest, m (ft)

If the upstream edge of a broad-crested weir is so rounded as to prevent contraction and if the slope of the crest is as great as the loss of head due to friction, flow will pass through critical depth at the weir crest; this gives the maximum C value of 1.70. For sharp corners on the broad crested weir, a minimum value of 1.44 should be used. Additional information on C values as a function of weir crest breadth and head is given in Table 10-2.

Table 10-2 Broad-crested weir coefficient C values as a function of weir crest breadth and head (coefficient has units of $m^{0.5}$/sec) (metric only)

Head[1] (m)	BREADTH OF CREST OF WEIR (m)														
	0.15	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.25	1.50	2.00	3.00	4.00
0.10	1.59	1.56	1.50	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.37	1.35	1.36	1.40	1.45
0.15	1.65	1.60	1.51	1.48	1.45	1.44	1.44	1.44	1.45	1.45	1.44	1.43	1.44	1.45	1.47
0.20	1.73	1.66	1.54	1.49	1.46	1.44	1.44	1.45	1.47	1.48	1.48	1.49	1.49	1.49	1.48
0.30	1.83	1.77	1.64	1.56	1.50	1.47	1.46	1.46	1.46	1.47	1.47	1.48	1.48	1.48	1.46
0.40	1.83	1.80	1.74	1.65	1.57	1.52	1.49	1.47	1.46	1.46	1.47	1.47	1.47	1.48	1.47
0.50	1.83	1.82	1.81	1.74	1.67	1.60	1.55	1.51	1.48	1.48	1.47	1.46	1.46	1.46	1.45
0.60	1.83	1.83	1.82	1.73	1.65	1.58	1.54	1.46	1.31	1.34	1.48	1.46	1.46	1.46	1.45
0.70	1.83	1.83	1.83	1.78	1.72	1.65	1.60	1.53	1.44	1.45	1.49	1.47	1.47	1.46	1.45
0.80	1.83	1.83	1.83	1.82	1.79	1.72	1.66	1.60	1.57	1.55	1.50	1.47	1.47	1.46	1.45
0.90	1.83	1.83	1.83	1.83	1.81	1.76	1.71	1.66	1.61	1.58	1.50	1.47	1.47	1.46	1.45
1.00	1.83	1.83	1.83	1.83	1.82	1.81	1.76	1.70	1.64	1.60	1.51	1.48	1.47	1.46	1.45
1.10	1.83	1.83	1.83	1.83	1.83	1.83	1.80	1.75	1.66	1.62	1.52	1.49	1.47	1.46	1.45
1.20	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.79	1.70	1.65	1.53	1.49	1.48	1.46	1.45
1.30	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.82	1.77	1.71	1.56	1.51	1.49	1.46	1.45
1.40	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.77	1.60	1.52	1.50	1.46	1.45
1.50	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.79	1.66	1.55	1.51	1.46	1.45
1.60	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.81	1.74	1.58	1.53	1.46	1.45

(1) Measured at least 2.5 H_c upstream of the weir

V-Notch Weir

The discharge through a v-notch weir is shown in Figure 10-14 and can be calculated from the following equation:

$$Q = 1.38 \tan(\theta / 2) H^{2.5} \quad \textbf{(10.23)}$$

where: Q = discharge, m^3/s (ft^3/s)
θ = angle of v-notch, degrees
H = head on apex of v-notch, m (ft)

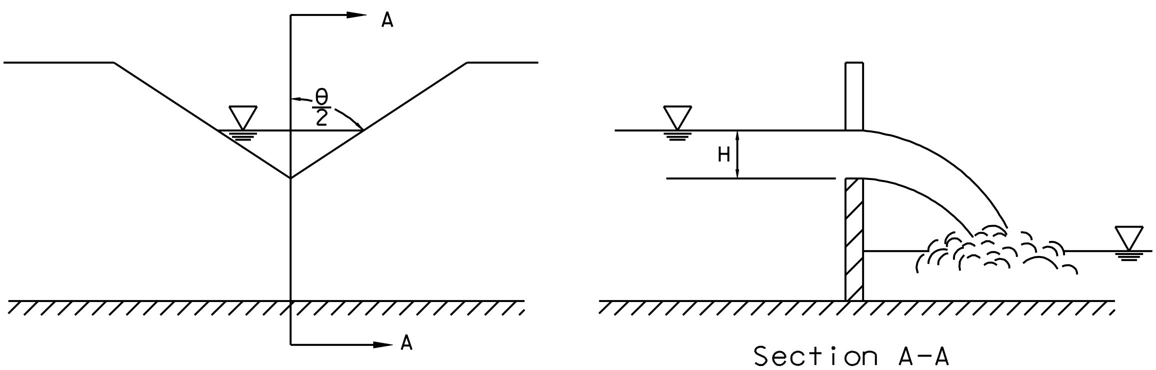

Figure 10-14 V-notch Weir

Proportional Weir

Although more complex to design and construct, a proportional weir may significantly reduce the required storage volume for a given site. The proportional weir is distinguished from other control devices by having a linear head-discharge relationship. This relationship is achieved by allowing the discharge area to vary nonlinearly with head.

Design equations for proportional weirs are as follows:

$$Q = 2.74 a^{0.5} b (H - a/3) \quad \textbf{(10.24)}$$

$$x/b = 1 - (0.315) [\arctan (y/a)^{0.5}] \quad \textbf{(10.25)}$$

where: Q = discharge, m^3/s (ft^3/s)
H = head above horizontal sill, m (ft)
Dimensions a, b, x, and y are as shown in Figure 10-15.